Dissipative Parameters in Ferrites and Insertion Losses in Waveguide Y-Circulators Below Resonance

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Abstract—Extensive microwave loss measurements have been performed at frequencies from 1.3 to 11 GHz on below-resonance waveguide Y circulators loaded with a wide variety of ferrite and garnet compositions.

Dissipative internal and external magnetic parameters have been measured on the same compositions. Also, dielectric loss measurements have been carried out. Two classes have been distinguished, defined by the following conditions: $\omega_M/\omega \leq 0.8$ and $0.85 \leq \omega_M/\omega \leq 1.05$. It is inferred that the (insertion loss) IL of such devices is independent of ΔH and mainly depends on the internal dissipative susceptibility χ_i'' and on the dielectric loss tan δ. The relation of the IL versus χ_i'' and tan δ in the case $\omega_M/\omega \leq 0.8$ is independent of frequency and given by the semiempirical equation $\text{IL} = 10 \log_{10} (1 - 2.85\,\chi_i'' - 1.60 \tan\delta - 0.017)^{-1}$.

I. Introduction

At microwave frequencies four dissipative parameters of a polycrystalline ferrite contribute *a priori* to the insertion loss (IL) of a microwave device. Three parameters are magnetic and one is electric. For reasons that are in some degree obvious but will be further clarified in what follows, it is important to specify how the four dissipative parameters are related to the overall losses in a waveguide Y circulator under specified operating conditions.

Only the resonance linewidth ΔH, the tangent of the dielectric loss angle tan δ, and sometimes the spin-wave linewidth ΔH_k, are indicated as the low-power microwave-loss parameters of ferrites by materials manufactures. It is now recognized that ΔH alone is by no means a suitable loss parameter: as an example, an X-band below-resonance Y circulator loaded with a Mg–Mn ferrite ($\Delta H \simeq 400$ Oe) shows the same very low insertion loss as a similar device loaded with YIG ($\Delta H \simeq 35$ Oe).

Rodrigue *et al.* [1] were able to correlate successfully the low-field low-power losses in a latching phase shifter with an intrinsic linewidth ΔH_i, derivable from the spin-wave linewidth of wave number k, ΔH_k.[1]

During a ferrite workshop at the 1969 Intermag Conference, Bosma emphasized the convenience of a new characterization of the significant physical parameters of ferrite materials as related to their device performance. Among other subjects, the significance of the ΔH parameter was discussed.

Stimulated by the lack of exhaustive data on the matter, we have undertaken the experimental work presented herein. To carry out our correlation, we have chosen the waveguide junction Y circulator operating at fields below resonance. The choice of the device type is mainly due to the following reasons: it is one of the most extensively employed ferrite devices in microwave techniques; the theoretical solution of the field distribution problem is not easily treatable. It is felt, therefore, that consistent experimental data can enlighten the dependence of device losses on the material's dissipative characteristics.

This work is presented as follows. In Section II those features of the tensor permeability parameters relevant for our device problem are reviewed, with emphasis on its dissipative components. In Section III, the experimental procedures and techniques are outlined, followed by the description of the results of measurements on the devices and in the resonant cavities.

A correlation between the different sets of data and the consequent indication of the important tensor elements are given, along with a critical discussion of the matter.

II. General Features of the Susceptibility Tensor

The tensor magnetic susceptibility of a polycrystalline ferrite in an external magnetic field H_0, directed along the Z axis is

$$\chi = \begin{pmatrix} \chi & jk & 0 \\ -jk & \chi & 0 \\ 0 & 0 & \chi_z \end{pmatrix} \tag{1}$$

where

$$\chi = \chi' - j\chi'' \qquad k = k' - jk'' \qquad \chi_z = \chi_z' - j\chi_z''. \tag{2}$$

The complex dielectric constant in the ferrite is defined by

$$\epsilon = \epsilon_r(1 - j \tan \delta) \tag{3}$$

where ϵ_r is the real part of the dielectric constant. When a volume element $d\tau$ of the ferrite is imbedded in an RF electromagnetic field of frequency $\nu = \omega/2\pi$,

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[1] The intrinsic magnetic loss is linearly related to ΔH_k, when originating from the doping of fast relaxers. Losses due to other mechanisms such as fine grains show a quite different dependence on ΔH_k (see, for example, [9]). For these reasons ΔH_k cannot be considered in all circumstances a measure of low-field losses.

$$h = h_x x + h_y y + h_y z$$
$$e = e_x x + e_y y + e_z z \tag{4}$$

and in a dc bias field

$$H_0 = H_0 z. \tag{5}$$

The total elementary volume losses dP are given by

$$\begin{aligned} dP &= dP_{\text{magnetic}} + dP_{\text{electric}} \\ &= \text{Re } (j\omega/2)[(\mathfrak{y}\cdot h)\cdot h^* - \epsilon^* \mid e \mid^2] d\tau \\ &= (\omega d\tau/2)[\chi''(\mid h_x \mid^2 + \mid h_y \mid^2) - 2k'' \text{ Im } (h_y h_x^*) \\ &\quad + \chi_z'' \mid h_z \mid^2 + \epsilon_r \mid e \mid^2 \tan \delta]. \end{aligned} \tag{6}$$

h^*, ϵ^*, and h_x^* are the complex conjugates of h, ϵ, and h_x, respectively.

In (6) we find the four dissipative parameters in the ferrite, χ'', k'', χ_z'', and $\tan \delta$. Inspection of (6) reveals that, owing to the alternative possible sign of k'' [2] and of Im $(h_y h_x^*)$, the term containing k'' may become negative in certain circumstances, thus possibly reducing the magnitude of the total magnetic dissipation.

This work is aimed at the determination of the respective influences of the dissipative parameters of the ferrite on the device losses. Therefore, it is appropriate to examine the connection between the tensor permeability parameters and the resonance linewidth ΔH.

The resonance linewidth is inversely proportional to the relaxation time of the uniform precessional mode of the spin waves. According to the phenomenological relaxation theories such as that of Landau–Lifshitz, the relaxation rate η and the gyromagnetic ratio γ are independent of H_0, thus providing a Lorentzian-shaped loss versus static field curve. In this case χ'' and k'' can be derived from ΔH [3] for every value of H_0 that magnetically saturates the sample.[2] However the relaxation rate (and, less appreciably, γ) are field-dependent quantities; i.e., ΔH can properly account for the magnetic losses only at fields near resonance. The study of the field dependence of the relaxation rate has led to the definition of an effective linewidth [4]–[6] $\Delta H_{\text{eff}} = 2\eta(H_0)/\gamma(H_0)$, which accounts for losses inside and far off the spin-wave manifold.

We have previously shown [7], [8] that ΔH_{eff} $(H_0 \to 0)$ is about eight times smaller than $\Delta H(H_0 = H_{\text{res}})$ in nickel ferrites at X band, and about from twenty to thirty times smaller in Mg–Mn ferrites at X band. Only on rare-earth-doped coarse-grained YIG compositions, ΔH_{eff} values have been found to be comparable to ΔH, while on fine-grained YIG, $\Delta H \gg \Delta H_{\text{eff}}$ [9].

These and other results make clear how misleading ΔH can be as a parameter for predicting below-resonance losses, thus suggesting a wider inspection on χ'', k'', and χ_z''. The following relations always hold in a polycrystalline ferrimagnetic body [2]:

$$\chi'', \chi_z'' \geq 0 \qquad \mid k'' \mid \leq \chi'' \tag{7}$$

[2] In the ferrite, in a Y circulator, regions exist where magnetic saturation is not attained.

to which, for the unmagnetized state,

$$\chi'' \equiv \chi_z'' \qquad k'' \equiv 0 \tag{8}$$

must be added, provided that the distribution of the local magnetization vector in the unmagnetized state is isotropic. From the phenomenological equations of motion [3] containing a damping parameter, the following relations can be derived:

$$\chi'' \simeq k'', \qquad \text{for } H_i \text{ close to } H_{i,\text{res}} \tag{9a}$$
$$\mid k'' \mid / \chi'' \to 0, \qquad \text{for } H_i \to 0 \tag{9b}$$
$$\chi_z'' \to 0, \qquad \text{for } H_i \to H_{i,\text{sat}} \tag{9c}$$

where H_i is the internal dc field, $H_{i,\text{res}}$ the internal dc field for ferromagnetic resonance, and $H_{i,\text{sat}}$ that for magnetization saturation.

It has been found experimentally that (9a) is approximately true [10], while (9b) holds under certain conditions given below. Equation (9c) is generally not true, owing to the inhomogeneities connected with local domain configurations. Schlömann has shown [11] that local inhomogeneities of the saturation magnetization (due to such causes as pores, nonmagnetic inclusions etc.) induce a torque upon the magnetization even if the average RF magnetic field $\langle h \rangle$ is parallel to the average magnetization $\langle 4\pi M \rangle$. For these reasons parallel pumping does not prevent a local Larmor precession excitation. Furthermore, a theory by Joseph and Schlömann [12] demonstrates the existence under certain circumstances of a *subthreshold parallel pump loss* independent of χ'' but contributing to the term containing χ_z''.

III. Experiments and Results

A. Measurement Procedures

In order to establish a correlation as unambiguous as possible between the losses in the circulators and the dissipative parameters in the ferrites, the latter were cavity-determined on spheres, rather than on disks.

Microwave measurements performed on disks provide the obvious advantage of parameter determination on samples with very similar geometry to the actual situation in a Y circulator.

On the other hand a comparison between measurements on disks and on spheres would show the following.

1) Measurements on disks are influenced by the diameter/thickness ratio [13].

2) Most likely, surface mechanical stresses due to machining and to the sintering process are responsible for alterations on the loss parameters [5], [14]; the more stress present, the greater the surface/volume ratio. The effects of such stresses are only reduced by annealing the samples.

3) Below-resonance imaginary susceptibility determinations require high measuring sensitivity. Disks would therefore require too large diameter sizes, appreciable fractions of the wavelength, and consequent nonuniformity of the RF field.

4) It is generally simpler to obtain spheres on which other microwave and physical determinations can be performed, such as the saturation magnetization and the gyromagnetic factor.

5) Spherical shape insures a uniform internal magnetic field if the sample is homogeneous. For the above reasons all susceptibility data are determined on spheres, in resonant microwave cavities.

Cavity measurements provide internal parameters directly related to the intrinsic properties of the ferrite or external parameters which depend both on the intrinsic properties and on the shape of the specimen.

The distinction between internal and external susceptibility has been pointed out by Spencer *et al.* [10] and will be indicated here by an i and e subscript.

In the quasi-static approximation the following relation holds [10]:

$$X_i = X_e/(1 - DX_e). \tag{10}$$

$D=\frac{1}{3}$ for spheres; X_i and X_e indicate $\chi \pm k$ or χ_z, depending on which of these parameters is being considered. χ_e'', $\chi_{e,z}''$, ΔH, and g_{eff} have been measured in rectangular TE_{10n} cavities, at the frequency at which the ILs. on circulators were previously determined. χ_e'' and k_e'' measurements required a circular polarization of the RF field; therefore a circular degenerate cavity oscillating in the TE_{112} mode at 9.35 GHz has been used. χ_i'' and k_i'' were derived from χ_e'' and k_e'' through (10). Tan δ values have been measured on small rods (1 mm diam, 15 mm long) in the center of a rectangular TE_{103} cavity at 9.4 GHz. Precision of the measurements is estimated to be about ± 20 percent in the degenerate cavity and about ± 10 percent in the other cavities.

The spheres for the dissipation measurements were obtained from the same triangles and disks on which IL measurements were performed. Some spheres, triangles, and disks were thermally annealed to reduce stresses, but only on spheres of fine-grained material was a noticeable reduction of the loss parameters observed.

Measurements in the Y circulators were performed on devices having the four different geometries outlined in Fig. 1. They are H-plane junction circulators: types A and C of Fig. 1 are narrow-band devices, while types B and D exhibit a broader bandwidth.

Most of the examined devices were X-band devices. The L- and S-band devices described in the following sections were constructed from some of the former ones by a scaling criterion as described in [15] and more explicitly in Section III-B. Care has been taken to keep the VSWR level always less than 1.10 and to insure the minimum obtainable IL. IL measurements were precise within ± 10 percent.

B. Results and Discussion

A large number of different ferrite compositions of the spinel and garnet type have been examined, all but one of which have been prepared in the Ceramic Labora-

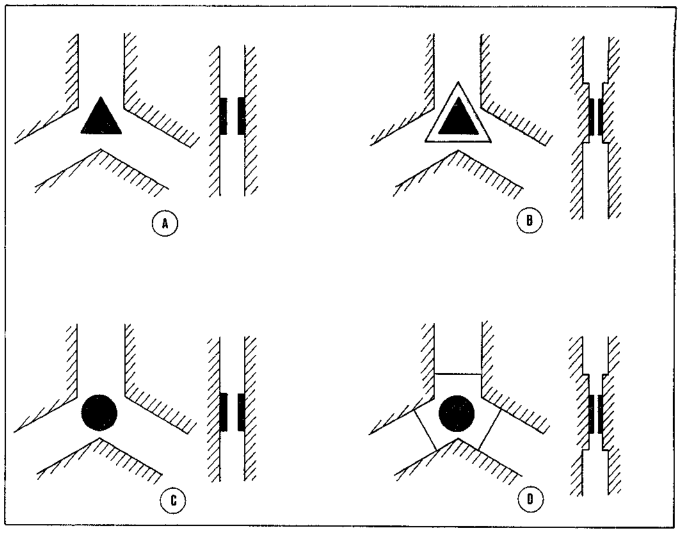

Fig. 1. Circulator and ferrite geometries described in this paper.

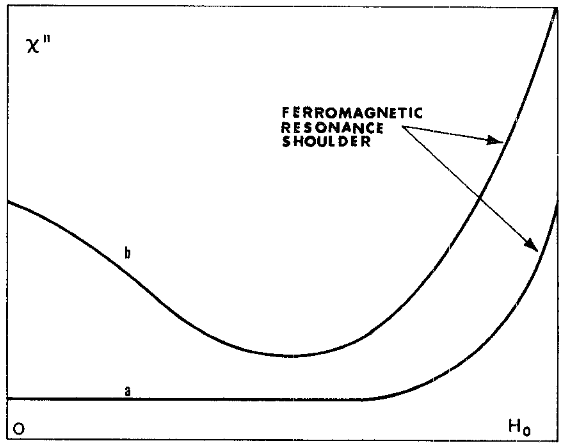

Fig. 2. Dissipative permeability χ'' versus external dc magnetic field H_0. Curve a, $\omega_M/\omega < 0.8$; curve b, $\omega_M/\omega > 0.8$.

tory of Selenia S.p.A. Very different saturation magnetizations (from 290 to 3100 G) have been included among the ferrite compositions. The microwave measurements have been performed both on the devices and the materials at different frequencies and have been divided into two classes: one that respects the condition $\omega_M/\omega < 0.80$ and the other that respects the condition $0.85 \leq \omega_M/\omega \leq 1.05$. The distinction follows the general criterion of avoiding low field losses [16] in a below-resonance device. The first classes generally respect this criterion.[3]

1) *Case of* $\omega_M/\omega \leq 0.8$: In Fig. 2 the normally encountered behavior of $\chi''(H_0)$ for $\omega_M/\omega \leq 0.8$ and $\omega_M/\omega > 0.8$ is shown in the region below resonance by curves a and b,[4] respectively. χ_z'' determinations were

[3] $\omega_M = \gamma 4\pi M_s$, where γ is the gyromagnetic factor. Polder and Smit [16] indicate in $\gamma(4\pi M_s + H_a)/\omega < 1$ the conduction for avoidance of low-field losses, where H_a is the anisotropic magnetic field. In this work H_a has not been considered. This can explain why $\omega_M/\omega \leq 0.8$ was found to be the condition for avoiding low-field losses in the samples examined in Fig. 3.

[4] Actually, in fine-grained ferrites, even with $\omega_M/\omega = 0.5$, a curve of type b was found in this laboratory [9].

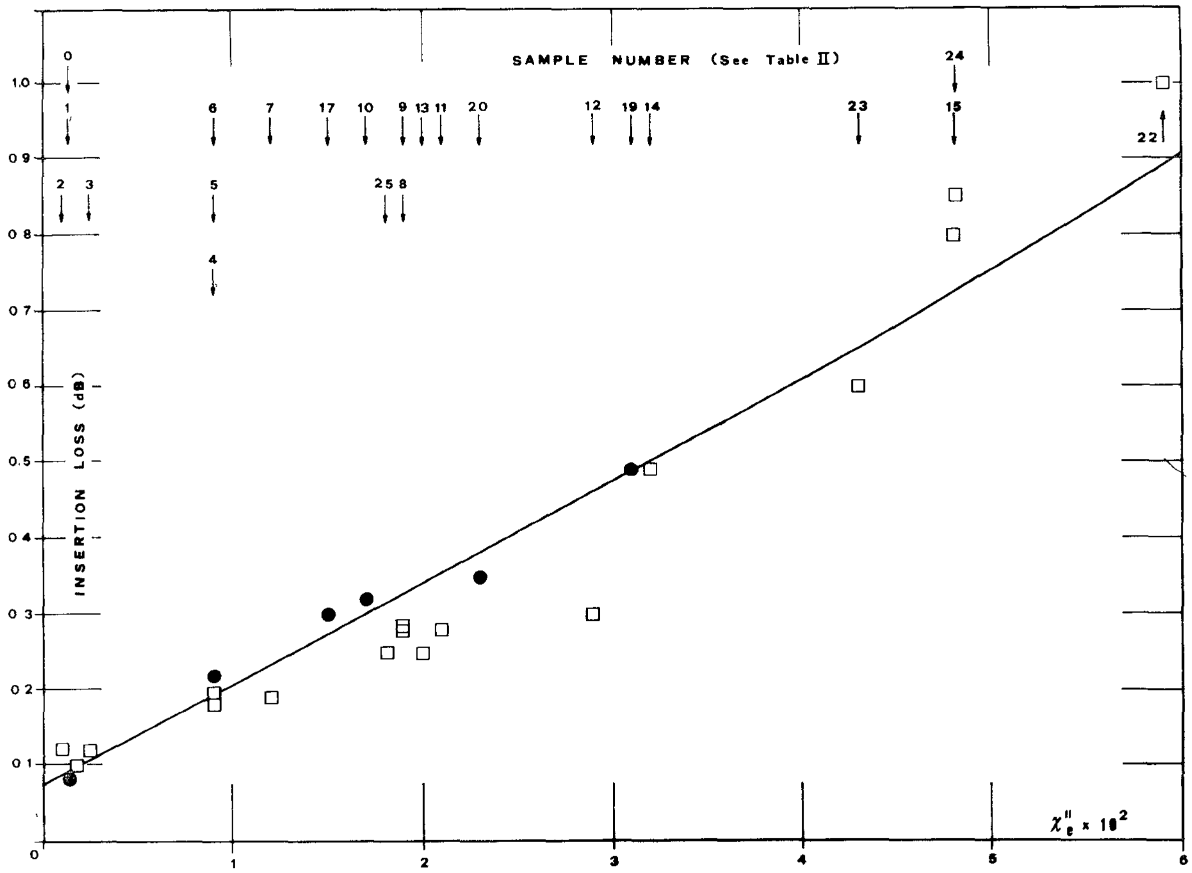

Fig. 3. Circulator insertion loss as function of χ_e'' for the case $\omega_M/\omega < 0.8$, at frequencies from 1.3 to 10.1 GHz. Sample number refers to the composition listed in Tables II and III. The continuous line is calculated following (16). Circles—measurements at 10.1 GHz on type B circulators. Squares—measurements at X, S, and L band on type A, B, C, and D circulators, as indicated in Table II.

started with the reasonable assumption that this parameter was only a minor contribution to the IL in the devices, mainly because of the essential orthogonality of $\boldsymbol{h}_{RF}$ with respect to $\boldsymbol{H}_0$. A preliminary check of this assumption was made on two Co-doped Mg–Mn ferrites (with 1-percent and 2-percent weight of CoO, respectively) each of them in the fine- and coarse-grained versions. It is well known that fine graining and Co^{2+}-doping both increase the magnetic losses and the high power thresholds. The four compositions all showed $\Delta H \simeq 400$ Oe, $g_{eff} = 2.0$, $4\pi M_s \simeq 2200$ G, and $\tan \delta < 2.5 \times 10^{-4}$. ILs were measured on a circulator (type B of Fig. 1) at 9.5 GHz, χ_e'' and $\chi_{e,z}''$ at 9.2 GHz. It was found that $\chi_{e,z}'' \simeq \chi_{i,z}''$ and is nearly constant in the approximate range $4\pi M_s/3 < H_0 < 4\pi M_s$. χ_e'' behaves in the same manner as curve a of Fig. 2, thus confirming the typical trend of the condition $\omega_M/\omega \lesssim 0.8$.

On the examined samples the magnitude of χ_z'' is about $\frac{1}{2}$ or $\frac{1}{3}$ of χ_e'' in conventionally sintered ferrites, while $\chi_e'' \simeq \chi_z''$ in fine-grained ferrites. The data of Table I show that, owing to $h_z \simeq 0$, χ_z'' is not related to the IL. The data in Fig. 3 and all subsequent measurements agree with this statement. χ_e'' data are reported, together with all the other relevant microwave and physical characterizations, in Fig. 3 and Tables II, and III.

TABLE I

COMPARISON OF χ_e'' AND $\chi_{e,z}''$ WITH IL AT 9.5 GHz

Ferrite Composition	$\chi_e'' \times 10^3$	$\chi_{e,z}'' \times 10^3$	IL (dB)
Mg, Mn (1-percent CoO), CG[a]	9	3.3	0.18
Mg, Mn (1-percent CoO), FG	12	7.5	0.19
Mg, Mn (2-percent CoO), CG	19	9	0.28
Mg, Mn (2-percent CoO), FG	21	18	0.28

[a] FG—fine grained; CG—coarse grained.

At this point it is useful to analyze (6) and the data of Fig. 3 more closely. The term $\chi_z'' \, h_z^2$ in (6), for the reasons explained above, can be neglected. Table III also shows that for the examined samples $k_i'' \ll \chi_i''$. Neglecting k_i'' with respect to χ_i'', the magnetic losses in a circulator can be written

$$P_{\text{magnetic}} \simeq (\omega/2) \int_F \chi_i''(h_x^2 + h_y^2) \, d\tau \tag{11}$$

where F indicates the volume of the ferrite. It is found experimentally that χ_i'' is approximately constant for fields $0 < H_0 < 8\pi M_s/3$ within the volume; therefore

$$P_{\text{magnetic}} \simeq (\chi_i''\omega/2) \int_F (h_x^2 + h_y^2) \, d\tau = A'\chi_i''. \tag{12}$$

TABLE II

Sample	Ferrite Composition	Type (Fig. 1)	Frequency (GHz)	$4\pi M_s$ (kG)	ΔH^a (Oe)	g^a	$\tan\delta \times 10^4$
0	Mg, Mn, Fe ferrite	B	9.9	2.20	430	2.04	<2.5
1	same as 0	B	10.1	2.20	430	2.04	<2.5
2	same as 0	B	9.5	2.20	430	2.04	<2.5
3	same as 0, FG	B	9.5	2.20	380	2.03	<2.5
4	Mg, Mn (1-percent Co), Fe ferrite	B	9.5	2.20	390	2.04	<2.5
5	same as 4	A	9.2	2.20	390	2.04	<2.5
6	same as 4	B	10.1	2.20	390	2.04	<2.5
7	Mg, Mn (1-percent Co), Fe ferrite, FG	B	9.5	2.20	320	2.04	<2.5
8	Mg, Mn (2-percent Co), Fe ferrite	B	9.5	2.20	460	2.03	<2.5
9	same as 8	A	9.2	2.20	460	2.03	<2.5
10	same as 8	B	10.1	2.20	460	2.03	<2.5
11	Mg, Mn (2-percent Co), Fe ferrite, FG	B	9.5	2.20	380	2.04	<2.5
12	Ni (Co) Al ferrite TT-2-115 (Trans. Tech., Inc)	C	9.2	1.60	330	2.45	10
13	Ni (Co, Cu) Al, Fe ferrite	D	8.9	1.70	190	2.33	16
14	Y, Gd, Al, In, Fe garnet	B	1.3	0.32	75	2.29	<2.5
15	Y, Gd, Dy, Al, In, Fe garnet	B	1.3	0.29	140	2.13	<2.5
16	Ni, Al, Fe ferrite. ID	B	10.1	1.70	730	2.56	160
17	same as 16, stoichiometric	B	10.1	1.85	600	2.48	100
18	same as 16, IE	D	10.1	1.95	500	2.43	270
19	Ni (Co) Al ferrite, ID	B	10.1	1.85	450	2.39	84
20	same as 19, stoichiometric	B	10.1	1.95	300	2.36	55
21	same as 19, IE	B	10.1	2.00	225	2.32	260
22	Y, Gd, Al, In iron garnet	B	1.3	0.34	105	2.29	<2.5
23	Y, Gd, Ho, Al, In iron garnet	B	1.3	0.29	115	2.35	<2.5
24	same as 15	D	1.25	0.29	140	2.13	<2.5
25	Y, Gd, iron garnet	B	3	0.75	160	2.15	<2.5

Note: This table lists some properties of materials on which IL and dissipation parameters were measured, for which $\omega_M/\omega < 0.8$. The IL and χ_e'' values, not reported here, are plotted in Fig. 3. The capital letter after each composition indicates the circulator type on which measurements were performed. FG—fine grained; ID—iron deficiency; IE—iron excess.

[a] ΔH and g values have been measured at 9.5 GHz, except for samples 14, 15, and 22–24, which were measured at 1.3 GHz, and sample 25 which was measured at 3 GHz.

TABLE III

Sample	$\chi_e'' \times 10^3$	$\kappa_e'' \times 10^3$	$\chi_i'' \times 10^3$	$\kappa_i'' \times 10^3$
8	18	10	14	3
11	22	12	20	1
13	20	7	18	−1
16	26	16	20	0
18	18	8	15	2

Note: This table lists degenerate cavity measurements on several ferrite samples for which the condition $\omega_M/\omega < 0.8$ holds. The sample number refers to those of Table II. Measurements were made at 9.35 GHz.

Similarly the dielectric losses are given by

$$P_{\text{dielectric}} = (\tan \delta\epsilon_r\omega/2) \int_F e^2\, d\tau = B' \tan \delta. \quad (13)$$

The losses in the waveguide are

$$P_c \simeq (\omega/8\sigma)^{1/2} \int_S h_x^2\, dS = C' \quad (14)$$

where S indicates the total metal surface and σ its electrical conductivity. If we denote with P_i the incident power, we have

$$\text{IL} = 10 \log_{10} [P_i/(P_i - P_{\text{magnetic}} - P_{\text{dielectric}} - P_c)]$$
$$\simeq 10 \log_{10} [(1 - A\chi_i'' - B \tan \delta - C)^{-1}]. \quad (15)$$

A, B, and C are derived from (12), (13), and (14) by normalizing to the incident power.

If the junction is scaled to operate from one frequency ω to another $\omega' = \omega/K$, the new volume element is $d\tau' = K^3 d\tau$, while the square of the normalized fields are divided for K^2. A and B do not change. The constant C depends on frequency as $\omega^{1/2}$.[5]

In this paper (see the data in Table II) the L-band circulators 15 and 24 and the S-band circulator, 25 have been scaled from the corresponding X-band circulators (0, 13, and 0, respectively), and subsequent measurements confirm the frequency independence of A and B.

In Table III degenerate cavity measurements have been performed on a part of the samples. The measured compositions have formulas and processing features that are representative of most of the X-band compositions listed in Table II, while their magnetic-loss levels are suitable for reaching high enough sensitivities in the degenerate cavity measurements.

It is seen that $\chi_i'' \simeq \chi_e''$. Reasonably assuming that this result holds for all the inspected samples, we have correlated (6) with the experimental points plotted in Fig. 3.

By imposing the experimental points related to samples 1 and 19 and by assuming $B \tan \delta = 0$, the values $C = 0.017$ and $A = 2.85$ are found. The points marked with full circles represent measurements performed on different ferrites in the same (type B) Y circulator and at the same frequency of 10.1 GHz. The following semi-empirical equation is obtained:

$$\text{IL} = -10 \log_{10} (1 - 2.85\, \chi_e'' - 0.017). \quad (16)$$

[5] The scaling criterion for a linear dimension L is $L' = KL$, while for the ferrite $\gamma' 4\pi M_s' = (\omega'/\omega)\, \gamma 4\pi M_s$.

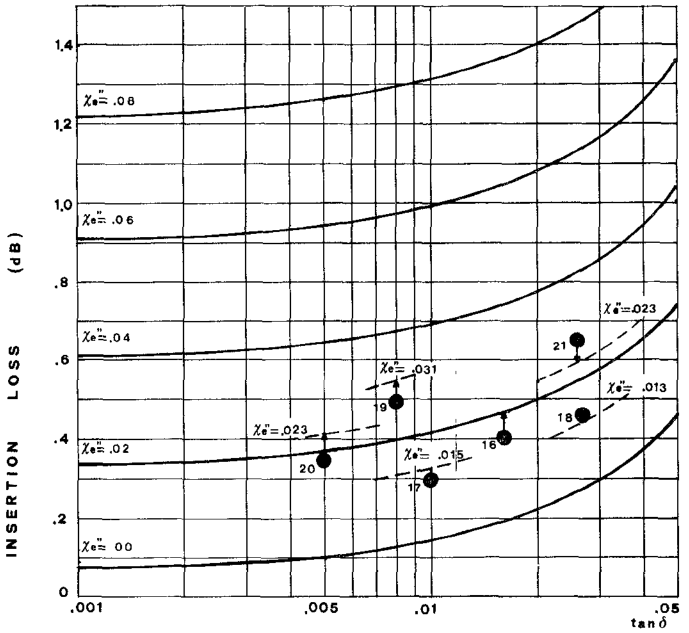

Fig. 4. Comparison between experimental points and theoretical curves of IL–tan δ correlation for different values of χ_e''. The curves follow (15); $A=2.85$, $B=1.60$, and $C=0.017$.

Equation (16) fits all of the experimental data very well at X band, S band, and L band independently of the circulators' geometries. C is kept constant in spite of its frequency dependence owing to its very low value.

The plot of Fig. 3 and the data of Tables II and III show the following.

a) The insertion loss is independent of ΔH.

b) The insertion loss is related to χ_e'' which in turn is very close to χ_i''.

c) χ_i'' is the significant parameter, as will be confirmed by the results of experiments for the case $0.85 \leq \omega_M/\omega \leq 1.05$ on nickel ferrites, while $\chi_i'' \gg k_i''$, in agreement with (9b).

d) The intercept value of the curve in Fig. 3 is 0.074 dB and is attributed to the losses in the walls of the waveguide.

It is concluded that the losses are described by ΔH_{eff} rather than by ΔH at resonance.

The agreement between (16) and the experimental points in Fig. 3 was obtained by neglecting tan δ. In order to evaluate the influence of the tan δ term on the overall losses, compositions 16–21 of Table II were examined. In Fig. 4 we have plotted the IL against tan δ for different values of χ_e''. The agreement between experimental points and the curves calculated from (15) is fairly good. B was determined by a best fit to experimental data and resulted in $B \simeq 1.6$.

It is seen that when tan $\delta < 5 \times 10^{-3}$, dielectric losses do not add much to the IL. This can be explained qualitatively, for the essentially planar structure, together with the relatively high real part of the dielectric constants (see Table II, 10–16), allows very small electric RF fields inside the ferrite.

2) *Case of* $0.85 \leq \omega_M/\omega \leq 1.05$: The measurements on the device were made on a circulator of type B (see Fig. 1) loaded with seven different types of nickel ferrites (see Table IV).

The ferrites were chosen to have different magnetic dissipation parameters by fine graining and Co^{2+} doping. The $\chi_e''(H_0)$ behavior follows that of curve b of Fig. 2. The static polarization field in the circulator was set in the region where χ_e'' is a minimum ($\chi_{e,\min}''$). χ_e'' measurements against the IL are plotted in Fig. 5. A certain dispersion of the experimental points is explained by the inhomogeneity of the internal magnetic dc field, as any small displacement in the field away from the minimum causes χ_e'' to increase. Experiments at two different measuring frequencies have been performed at 9.2 and 11.0 GHz. The phenomenological curve of Fig. 3 has been also drawn here for comparison.

A comparison of Fig. 3 with Fig. 5 and with Table IV leads to the following observations.

a) When $0.85 \leq \omega_M/\omega \leq 1.05$, the IL and χ_e'' are strongly frequency dependent.

b) When $\omega_M/\omega \simeq 1.05$ at a fixed IL value, χ_e'' is from two to three times larger than for the $\omega_M/\omega < 0.8$ case.

c) As ω_M/ω decreases, the IL versus χ_e'' curves approach the $\omega_M/\omega < 0.8$ curve.

A possible explanation is given by inspection of the degenerate cavity measurements. The data in Table IV show that $2\chi_i'' \simeq \chi_e''$; therefore, if χ_i'' is to be substituted for χ_e'', the IL versus χ_i'' points would lie close to the phenomenological curve drawn in Fig. 3, thus suggesting that χ_i'' is the significant loss parameter. The result is not surprising if the ferrite geometry, much closer to a thin disk rather than to a sphere, is considered.

IV. Comments

Finally, we should like to emphasize that the essential results of our work, in our opinion, retain their validity for every linear ferrite device operating below resonance, such as the strip-line junction circulator and the nonreciprocal latching phase shifter.

To clarify such a statement, our results are compared with those of Fay and Comstock on the strip-line circulator [17] and of Rodrigue *et al.* on the digital phase shifter [1].

In the case of the strip-line circulator it was found that

$$\text{IL} = 10 \log_{10} (1 - Q_L/Q_0)^{-1} \tag{17}$$

where Q_L is the external Q of the circulator and Q_0, neglecting losses in the metal, is given by $Q_0^{-1} = \mu_{\text{eff}}''/\mu_{\text{eff}}' + \tan \delta$. By assuming a conventional Lorentzian model at low-bias field values for $\omega_M/\omega \ll 1$, Fay and Comstock find that

$$\mu_{\text{eff}}'/\mu_{\text{eff}}'' \simeq 2\omega^2/\gamma^2 4\pi M_s \Delta H. \tag{18}$$

If, instead of ΔH, ΔH_{eff} is substituted in (18), and noting that $2\omega^2/\gamma^2 4\pi M_s \Delta H_{\text{eff}} = 1/\chi_e''$, the following ex-

TABLE IV

Sample	Composition	$4\pi M_s$ (G)	ΔH (Oe)	g	$\tan\delta\times10^4$	$\chi_e''\times10^3$ $H_0=0$ 9 GHz	11 GHz	Degenerate Cavity Measurements (at 9.35 GHz; $H_0=1400$ Oe) $\chi_e''\times10^3$	$\kappa_e''\times10^3$	$\chi_i''\times10^3$	$\kappa_i''\times10^3$
1	Ni ferrite	3000	450	2.29	<2.5	315	33				
2	Ni ferrite	3000	440	2.29	<2.5	200	34				
3	Ni ferrite	3100	500	2.25	20	570	25	35	25	17	0
4	Ni ferrite hot pressed	2900	450	2.35	4	230	68	115	85	60	10
5	Ni ferrite hot pressed	3000	550	2.25	<2.5	550	39				
6	Ni(Co)ferrite	3000	200	2.16	<2.5	110	35				
7	Ni(Co)ferrite	3000	225	2.25	<2.5	86	30	43	37	20	5

Note: This table lists the physical and microwave parameters on several Ni and Ni(Co) ferrites. The sample numbers correspond to those of Fig. 5. ΔH and g were measured at 9.5 GHz, $\tan\delta$ at 9.4 GHz.

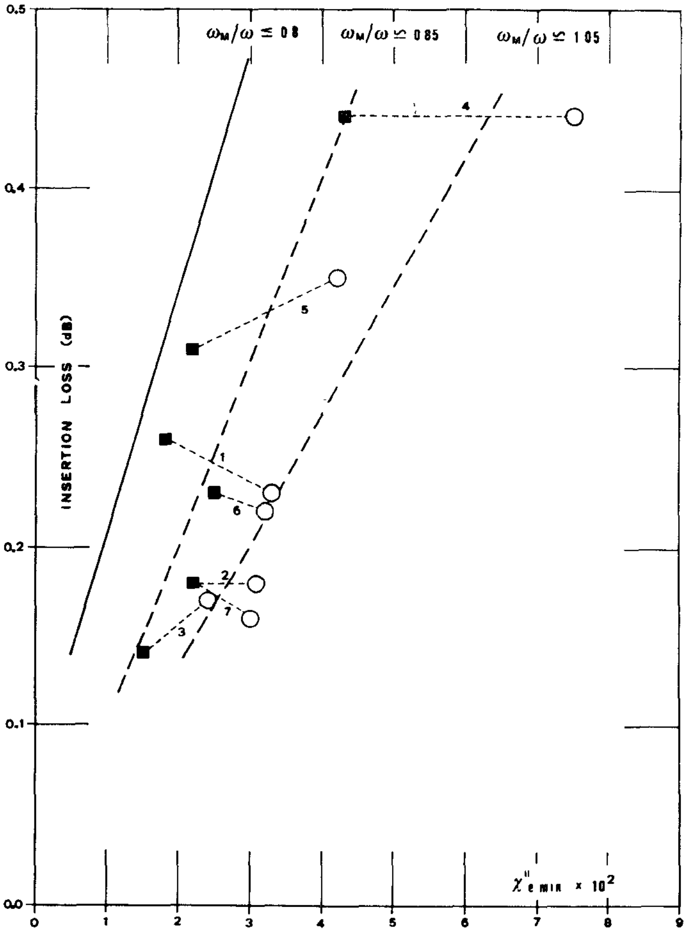

Fig. 5. Circulator insertion loss as function of $\chi_e''_{min}$ for the case $0.85\leq\omega_M/\omega\leq1.05$ at two different frequencies on a series of Ni(Co) ferrites and coarse- and fine-grained Ni ferrites. For the other properties of these materials see Table IV. Small dashed lines connecting pairs refer to the same samples. Two large dashed lines refer to samples for which $\omega_M/\omega\cong0.85$ and $\omega_M/\omega\simeq1$ as indicated. The solid line is the same as in Fig. 3.

pression is obtained:

$$\mathrm{IL} = 10 \log_{10} [1 - Q_L(\chi_e'' + \tan\delta)]^{-1} \qquad (19)$$

which is quite similar to (15).

Equation (19) shows a dependence of the IL through Q_L on the geometry of the circulator, as the coefficients A, B, and C do in (15). This dependence is very weak in our case. It is found that the IL remains approximately constant if the same ferrite disks or triangles are transferred from the circulators of types A and C (narrow bandwidth) to those of types B and D (broader bandwidth) of Fig. 1, respectively.

In the case of the latching phase shifter, Rodrigue *et al.* express the intrinsic losses in terms of an intrinsic linewidth ΔH_i, which is "obtained from single crystal data on material of equal quality and/or from parallel pump measurements of spin-wave linewidth ΔH_k [1]." It is apparent that ΔH_i is basically the same as ΔH_{eff}. For a given polarization the IL/360° of differential phase shift is linear in ΔH_i in much the same way as the IL is linear in χ_e'' in our case (Fig. 3). It must be pointed out that while the dependence of the IL on ΔH_{eff} is linear only if the ratio $\omega^2/\gamma\omega_M$ is kept constant, the IL varies linearly with χ_e'' independently of the above ratio, provided $\omega_M/\omega<0.8$. Furthermore, at least for sufficiently low values of ΔH_i, the IL/360° does not seem to depend any more on the magnetic polarization sign in the toroid, in perfect agreement with our finding that k'' is negligible when $\omega_M/\omega<0.8$.

Another reason for the good agreement between Rodrigue *et al.* and the authors is the following: for $\omega_M/\omega<0.8$, the χ_e'' versus H_0 curve behaves as curve *a* of Fig. 2. This means that at low fields for a given frequency, only one loss parameter (ΔH_{eff}, χ_e'' or ΔH_i) is sufficient to individuate the magnetic losses in the material.

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Analytical and Numerical Studies of the Relative Convergence Phenomenon Arising in the Solution of an Integral Equation by the Moment Method

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Abstract—**The relative convergence phenomenon that occurs in the numerical solution of the integral equation for the iris discontinuity problem is studied both analytically and numerically. It is shown that the solution for the aperture field can be highly dependent upon the manner in which the kernel and the unknown function are approximated in the process of constructing a matrix equation by the moment method. An analytical explanation is provided for the above phenomenon and the theoretical predictions are verified numerically. Also included is a suggested numerical algorithm for detecting and alleviating the relative convergence behavior for more general problems.**

I. Introduction

IT WAS pointed out a number of years ago [1] that the manner in which one partitions a doubly infinite matrix arising in the formulation of the boundary value problem in a bifurcated waveguide significantly affects the results for the scattering coefficients in the guide. This phenomenon was referred to as "relative convergence" and it was proven that there exists a unique choice for the partitioning ratio that yields the correct result. It was also demonstrated that anything but the correct choice of partitioning will lead to results that violate the "edge condition" [2].

Recently it was discovered that the relative convergence phenomenon also occurs in a variety of other problems [3]–[5], even where a completely different mode of formulation is employed. It was found, for instance, that the matrix equation obtained by the mode matching formulation [6], [7] or by the application of the moment method to integral equations exhibits relative convergence. As an example, it has been found that the integral equation

$$\int_0^b K(x, x')\psi(x')dx' = g(x), \qquad 0 < x < b \qquad (1)$$

for the iris discontinuity problem in a waveguide ex-

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